

# Detecting multi-spin interaction of an XY spin chain by geometric phase of a coupled qubit

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## Abstract

We investigate geometric phase (GP) of a qubit symmetrically coupled to a XY spin chain with three-spin interaction in a transverse magnetic field. An analytical expression for the GP is found in the weak coupling limit. It is shown that the GP displays a sharp peak or dip around the quantum phase transition (QPT) point of the spin chain. Without the three-spin interaction, the GP has a peak or dip around the critical point  $\lambda = 1$ . If the three-spin interaction exists, the peak or dip position is obviously shifted away from the original position. This result reveals that the GP may be taken as an observable to detect both the existence and strength of multi-spin interaction in a spin chain.

*Keywords:* geometric phase; spin chain; multi-spin interaction

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## 1. Introduction

Geometric phases (GPs) have been proposed as a typical mechanism for a quantum system to keep the memory of its evolution in Hilbert space. Since discovered first by Berry [1], GPs have become an object of intense research in both the theoretical and experimental realm [2]. The original concept of GPs has been generalized to the non-adiabatic case [3], the noncyclic case [4] and the degenerate state [5]. It is also noticed that GPs can be related to a number of important phenomena in physics [6] such as the Aharonov-Bohm

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effect [7], the quantum Hall effect [8] and even applications in the quantum information processing [9, 10, 11, 12].

In recent years, relation between GPs and quantum phase transitions (QPTs) [13] in various closed many-body systems has attracted many interests [14, 15, 16, 17]. Carollo *et al.* [14] showed that GPs are much sensitive to controlling parameters of spin chains and can be exploited as a tool to detect critical regions of the systems. Zhu [15] investigated GPs in the XY spin chain and showed that the ground-state GP obeys the scaling behavior in the vicinity of QPTs. And it is also shown that the connection of GPs with the typical features of QPTs such as the scaling behavior, critical exponents and so on is not restricted to the XY spin chain model but universal for quantum many-body systems [15].

Since quantum systems are unavoidably to interact with their surroundings, the time evolution of their states is generally nonunitary. Therefore, it is desirable to extend the concept of GPs from closed quantum systems to open ones. Until now, many approaches have been proposed for this purpose [18, 19, 20, 21, 22]. The first work concerned with this point was given in Ref. [19], where GP is investigated purely as a mathematical problem. Based on the experimental context of quantum interferometry, Sjöqvist *et al.* [20] introduced a definition of GP for mixed states undergoing unitary evolution. Tong and coworkers [22] developed a kinematic approach to GP for open quantum systems in nonunitary evolution led by the environments. According to the definition of GP for open systems, many works concerned with the correction of environments to GPs have been done [23, 24, 25, 26, 27]. Yuan *et al.* [24] studied GP of a qubit coupled to an antiferromagnetic spin and found that the GP changes abruptly to zero when the spin chain undergoes a spin-flop transition. Lombardo *et al.* [25] computed the GP of a spin-1/2 linearly coupled to a harmonic oscillator reservoir at arbitrary temperatures and estimated the time scale for experimentally measuring the GP. Recently, Villar *et al.* [26] investigated GP of a spin-1/2 particle in the presence of a composite environment, composed of an external bath and another spin-1/2 particle. Their results show that the initial entanglement enhances the sturdiness of GP to decoherence. Cuchietti *et al.* [27] reported a measurement of GP for a spin-1/2 undergoing nonunitary evolution induced by coupling of an environment with a NMR quantum simulator.

In previous studies of GPs with spin environments, only is the nearest-neighbor spin interaction considered [24, 28]. Besides the two-spin kind of interactions, however, multi-spin interactions may also exist in spin chains

[29, 30, 31, 32]. As a result, phase transition points may be changed by the multi-spin interaction. In previous investigations, it has been shown that GPs of a qubit coupled to a spin chain change greatly around phase transition points of the spin chain and can signal out the appearance of phase transitions. According to this point, GPs are expected as a detector of phase transitions of quantum many-body systems [14, 15, 16, 17]. With the same reason, we expect that GPs of a qubit coupled to a spin chain with multi-spin interaction may be used as an observable to detect the multi-spin interaction. In the present work, we shall investigate GPs of a qubit symmetrically coupled to an anisotropic XY spin chain, in which a three-spin interaction is included besides the nearest-neighbor spin interaction. According to the definition of GP given in Ref. [22], we obtain an analytical expression for the GP of the central qubit in the weak coupling limit. Our results show that the variation of GP at the critical points of quantum phase transitions is much sensitive to the three-spin interaction, and the GP can be taken as a tool to detect the existence and strength of multi-spin interaction in spin chains.

The present paper is organized as follows. In Section 2, the model is introduced. In Section 3, an analytical expression of the GP is obtained in the weak coupling limit and the effect of the multi-spin interaction on the GP is investigated. Finally, a brief summary is given in Section 4.

## 2. The Model

The model under consideration is composed of a central spin-1/2 (or qubit) and a  $N$ -spin-1/2 chain. The central spin is symmetrically and transversely coupled to the circle spin chain, in which besides the nearest-neighbor spin interaction a three-spin interaction is also included. Meanwhile, a transverse magnetic field is homogeneously applied to each spin of the chain. The associated Hamiltonian reads

$$\begin{aligned}
H = & \eta\sigma_0^z - \sum_{l=1}^N \left[ \frac{(1+\gamma)}{2}\sigma_l^x\sigma_{l+1}^x + \frac{(1-\gamma)}{2}\sigma_l^y\sigma_{l+1}^y + \lambda\sigma_l^z \right] \\
& - \alpha \sum_{l=1}^N (\sigma_{l-1}^x\sigma_l^z\sigma_{l+1}^x + \sigma_{l-1}^y\sigma_l^z\sigma_{l+1}^y) - g\sigma_0^z \sum_{l=1}^N \sigma_l^z, \quad (1)
\end{aligned}$$

where  $\eta$  is the transition frequency between the ground state ( $|g\rangle$ ) and the excited state ( $|e\rangle$ ) of the central qubit,  $\gamma$  and  $\alpha$  are the anisotropic parameter

of the two-spin interaction and the strength of the three-spin interaction, respectively,  $\lambda$  is the coupling constant of the spin chain with the transverse magnetic field, and  $g$  is the coupling strength between the central qubit and the spin circle. In (1),  $\sigma_l^m (m = x, y, z)$  are the Pauli matrices for spin at the  $l$ th site of the spin chain and  $\sigma_0^z (= |e\rangle\langle e| - |g\rangle\langle g|)$  is the population inversion operator for the central spin.

The free-motion term of the central qubit in (1) can be removed by the rotating transformation  $\exp(-i\eta\sigma_0^z t)$ . Then, following the approach proposed in Ref. [13, 32], the rotated Hamiltonian can be fully diagonalized. Since  $[\sigma_0^z, \sigma_l^m] = 0$ , an operator-valued parameter  $\Lambda = \lambda + g\sigma_0^z$  is a conserved quantity. Thus  $\Lambda$  can be treated as a  $c$  number during diagonalizing the Hamiltonian. Obviously,  $\Lambda$  has two eigenvalues  $\Lambda_n = \lambda + (-1)^n g$  with  $n = 0, 1$ .

By introducing the Jordan-Wigner transformation [13]

$$\sigma_l^x = \prod_{i<l} (1 - 2c_i^\dagger c_i) (c_l + c_l^\dagger), \quad (2)$$

$$\sigma_l^y = -i \prod_{i<l} (1 - 2c_i^\dagger c_i) (c_l - c_l^\dagger), \quad (3)$$

$$\sigma_l^z = 1 - 2c_l^\dagger c_l, \quad (4)$$

where  $c_l^\dagger$  and  $c_l$  are the mapped spinless and fermionic creation and annihilation operators. Substituting Eqs. (2)-(4) into (1), one obtains

$$\begin{aligned} H = & - \sum_{l=1}^N \left\{ \left[ c_l^\dagger c_{l+1} + c_{l+1}^\dagger c_l + \gamma (c_l^\dagger c_{l+1}^\dagger - c_l c_{l+1}) \right] \right. \\ & \left. + 2\alpha (c_{l-1}^\dagger c_{l+1} + c_{l+1}^\dagger c_{l-1}) + \lambda (1 - 2c_l^\dagger c_l) \right\}. \end{aligned} \quad (5)$$

Then by means of the Fourier transformation [13]

$$c_l = -\frac{1}{\sqrt{N}} \sum_{k=-M}^M e^{i2\pi kl/N} d_k, \quad (6)$$

$$c_l^\dagger = -\frac{1}{\sqrt{N}} \sum_{k=-M}^M e^{-i2\pi kl/N} d_k^\dagger, \quad (7)$$

with  $M = N/2$  for  $N$  even and  $M = (N - 1)/2$  for  $N$  odd, one gets

$$H = \sum_{k>0} 2(\Lambda - \cos ka - 2\alpha \cos 2ka) d_k^\dagger d_k + i\gamma \sin ka (d_k^\dagger d_{-k}^\dagger - d_{-k} d_k), \quad (8)$$

where  $d_k$  and  $d_k^\dagger$  are the fermionic annihilation and creation operators in the momentum space, respectively.

In order to diagonalize the Hamiltonian (8), we introduce the Bogoliubov transformation [13]

$$\gamma_{k,\Lambda} = \cos \frac{\theta_k^\Lambda}{2} d_k - i \sin \frac{\theta_k^\Lambda}{2} d_{-k}^\dagger. \quad (9)$$

By substituting (9) into (8), the Hamiltonian (1) can be fully diagonalized and written in the form

$$H = \sum_{k>0} \Omega_{k,\Lambda} \left( \gamma_{k,\Lambda}^\dagger \gamma_{k,\Lambda} - \frac{1}{2} \right), \quad (10)$$

where

$$\Omega_{k,\Lambda} = 2\sqrt{(\Lambda - \cos ka - 2\alpha \cos 2ka)^2 + \gamma^2 \sin^2 ka}, \quad (11)$$

and

$$\theta_k^\Lambda = \arctan \left( \frac{\gamma \sin ka}{\Lambda - \cos ka - 2\alpha \cos 2ka} \right) \quad (12)$$

with  $a = 2\pi/N$ .

It is noted that the fully diagonalized Hamiltonian for the pure spin chain can be obtained by setting  $g = 0$  in Eq. (10). Correspondingly, the energy spectrum  $\Omega_{k,\lambda}$ , the parameter  $\theta_k^\lambda$  and the mode operator  $\gamma_{k,\lambda}$  for the pure spin chain can be obtained just by changing  $\Lambda$  into  $\lambda$  in Eqs. (9)-(12). It can be easily proved that both the mode operators  $\gamma_{k,\lambda}$  for the spin chain and  $\gamma_{k,\Lambda}$  for the qubit-chain coupled system are related by

$$\gamma_{k,\Lambda} = \gamma_{k,\lambda} \cos \alpha_{k,\Lambda} - i\gamma_{-k,\lambda}^\dagger \sin \alpha_{k,\Lambda} \quad (13)$$

with  $\alpha_{k,\Lambda} = (\theta_k^\Lambda - \theta_k^\lambda)/2$ .

### 3. Geometric Phase of the Central Qubit

Let us suppose that at  $t = 0$  the central qubit is decoupled from the spin environment, and the qubit is in a state

$$|\phi(0)\rangle = \cos \frac{\beta}{2} |e\rangle + \sin \frac{\beta}{2} |g\rangle. \quad (14)$$

The ground state of the spin chain is defined as  $\gamma_{k,\lambda} |G\rangle_\lambda = 0$ . It can be written in the form  $|G\rangle_\lambda = \prod_{k>0} \left( \cos \frac{\theta_k^\lambda}{2} |0\rangle_k |0\rangle_{-k} + i \sin \frac{\theta_k^\lambda}{2} |1\rangle_k |1\rangle_{-k} \right)$ , where  $|0\rangle_k$  and  $|1\rangle_k$  are the vacuum and single excitation states of the  $k$ th pure environment mode, respectively. Then according to (13),  $|G\rangle_\lambda$  can be rewritten as

$$|G\rangle_\lambda = \prod_{k>0} \left( \cos \alpha_{k,\Lambda} - i \sin \alpha_{k,\Lambda} \gamma_{k,\Lambda}^\dagger \gamma_{-k,\Lambda}^\dagger \right) |G\rangle_\Lambda, \quad (15)$$

where  $|G\rangle_\Lambda$  is the ground state of the Hamiltonian (10) and satisfies  $\gamma_{k,\Lambda} |G\rangle_\Lambda = 0$ .

At time  $t$ , the qubit-environment coupled system unitarily evolves into the state  $\rho(t) = U(t) (|\phi(0)\rangle \otimes |G\rangle_{\lambda\lambda} \langle G| \otimes \langle \phi(0)|) U^\dagger(t)$  with the time evolution operator  $U(t) = \exp(-iHt)$ . By tracing  $\rho(t)$  over variables of the spin chain, we obtain the reduced density matrix for the central qubit

$$\rho_s(t) = \begin{pmatrix} \cos^2 \frac{\beta}{2} & \frac{1}{2} \sin \beta F(t) \\ \frac{1}{2} \sin \beta F^*(t) & \sin^2 \frac{\beta}{2} \end{pmatrix}, \quad (16)$$

where  $F(t)$  is the decoherence factor and is given by

$$F(t) = {}_\lambda \langle G | U_{\Lambda_1}^\dagger(t) U_{\Lambda_0}(t) | G \rangle_\lambda. \quad (17)$$

The evolution operators  $U_{\Lambda_0}(t)$  and  $U_{\Lambda_1}(t)$  are obtained from  $U(t)$  by replacing  $\Lambda$  with  $\lambda + g$  and  $\lambda - g$ , respectively. From Eqs. (15) and (17), we can work out the explicit expression for the modulus of the decoherence factor

$$|F(t)| = \prod_{k>0} \left\{ AB \cos(\Omega_{k,\Lambda_0} - \Omega_{k,\Lambda_1})t - AB^2 \sin^2(\alpha_{k,\Lambda_0} - \alpha_{k,\Lambda_1}) \right. \quad (18) \\ \left. + 1 - \sin^2(2\alpha_{k,\Lambda_0}) \sin^2(\Omega_{k,\Lambda_0}t) - \sin^2(2\alpha_{k,\Lambda_1}) \sin^2(\Omega_{k,\Lambda_1}t) \right\}^{1/2}$$

where  $A = \sin(2\alpha_{k,\Lambda_0}) \sin(2\alpha_{k,\Lambda_1})$ ,  $B = 2 \sin(\Omega_{k,\Lambda_0}t) \sin(\Omega_{k,\Lambda_1}t)$ .

According to the GP definition for an open system, which is proposed in Ref. [22], the GP acquired by the qubit in a quasi period is given by

$$\Phi = \arg \left( \sum_k \sqrt{\varepsilon_k(0) \varepsilon_k(T)} \langle \varepsilon_k(0) | \varepsilon_k(T) \rangle e^{-\int_0^T \langle \varepsilon_k(t) | \partial / \partial t | \varepsilon_k(t) \rangle dt} \right), \quad (19)$$

where  $\varepsilon_k(t)$  and  $|\varepsilon_k(t)\rangle$  are the eigenvalues and the corresponding eigenstates of the reduced density matrix (16), and  $T = 2\pi/\eta$  is the time evolution cycle of the qubit when it is isolated from the environment. If the qubit is coupled to the spin chain, the evolution of its state is no more periodic. However, if the coupling is weak, we may consider a quasi cyclic path  $P : t \in [0, T]$ .

The instantaneous eigenvalues and the corresponding eigenvectors of (16) are found to be

$$\varepsilon_{\pm}(t) = \frac{1}{2} \left( 1 \pm \sqrt{\cos^2 \beta + \sin^2 \beta |F(t)|^2} \right) \quad (20)$$

$$|\varepsilon_{\pm}(t)\rangle = e^{-i\eta t} \sin \frac{\beta_{\pm}(t)}{2} |e\rangle + \cos \frac{\beta_{\pm}(t)}{2} |g\rangle, \quad (21)$$

with

$$\beta_{\pm}(t) = 2 \arctan \frac{\cot \beta \pm \sqrt{\cot^2 \beta + |F(t)|^2}}{|F(t)|}. \quad (22)$$

Since  $\varepsilon_-(0) = 0$ , Eq. (19) shows that only the eigenvalue  $\varepsilon_+(t)$  and corresponding eigenvector  $|\varepsilon_+(t)\rangle$  have contribution to the GP of the qubit. Upon substituting Eqs. (19)-(22) into Eq. (19), we obtain the GP of the central qubit

$$\Phi = \eta \int_0^{2\pi/\eta} dt \sin^2 \frac{\beta_+}{2}. \quad (23)$$

If the qubit has no interaction with the spin chain, i.e.  $g = 0$ , the well-known result  $\Phi = \pi(1 + \cos \beta)$  can be recovered from (23). When the qubit is coupled to the spin chain, the GP will be modified. In order to get a basic and clear intuition about the effect of the three-spin interaction on the GP acquired by the qubit in a quasi cycle, we first make an approximate analysis on the GP.

From Eqs. (22) and (23), we see that the correction of the spin chain to the GP is completely included in the decoherence factor  $F(t)$ . Therefore, we first analyze the decoherence factor. For this purpose, following Refs. [33],

we introduce a cutoff number  $K_c$ , which determines the largest energy scale of the spin chain, and define the partial product

$$|F(t)|_c = \prod_{k>0}^{K_c} F_k \geq |F(t)|, \quad (24)$$

and the corresponding partial sum  $S(t) = \ln |F(t)|_c = \sum_{k>0}^{K_c} |\ln F_k|$ . For small  $k$ , we can expand (11) as a power series of  $k$  and obtain

$$\Omega_{k,\Lambda_n} = 2 |\Lambda_n - 1 - 2\alpha| \quad (25)$$

and

$$\sin \alpha_{k,\Lambda_n} \approx \frac{(-1)^{n+1} 2\pi\gamma k g}{N |(\Lambda_n - 1 - 2\alpha)(\lambda - 1 - 2\alpha)|}, \quad (26)$$

$$\sin(\alpha_{k,\Lambda_0} - \alpha_{k,\Lambda_1}) \approx \frac{-2\pi\gamma k g}{N |(\Lambda_0 - 1 - 2\alpha)(\Lambda_1 - 1 - 2\alpha)|}. \quad (27)$$

In this way, the partial sum  $S(t)$  can approximately be written as

$$S(t) \approx -\frac{1}{2} \frac{(2\pi\gamma g)^2}{N^2 (\lambda - 1 - 2\alpha)^2} \sum_{k>0}^{K_c} k^2. \quad (28)$$

If  $K_c$  is small, in the thermal dynamical limit  $N \rightarrow \infty$ , one may ignore all the terms related to  $N^{-l}$  with  $l > 3$  in (28). Consequently, in a short time, we have

$$|F(t)|_c \approx e^{-\tau t^2}, \quad (29)$$

where  $\tau = 8E(K_c)\gamma^2 g^2 / (\lambda - 1 - 2\alpha)^2$  and  $E(K_c) = 4\pi^2 K_c(K_c + 1) \times (2K_c + 1) / (6N^2)$ . This result shows that the decoherence factor would decay in a Gaussian-type behaviour in a short time. Compared with that in the Ising spin environment, the decay rate  $\tau$  is modulated by the three-spin interaction  $\alpha$  of the chain [33].

Upon substituting (29) into (22), keeping all the terms up to the second order of the coupling strength  $g$  and completing the integral (23), we obtain the approximate expression for the GP

$$\Phi = \pi(1 + \cos \beta) + \frac{64E(K_c)\pi^3\gamma^2 \cos \beta \sin^2 \beta}{3\eta^2 (\lambda - 1 - 2\alpha)^2} g^2. \quad (30)$$



In the above expression, the first term comes out when the qubit is not coupled to the spin chain and undergoes a unitary evolution. The second term is the modification induced by the spin chain. Since the modification is proportional to the squared coupling strength  $g$ , and the squared reciprocal of the transition frequency  $\eta$  and the criticality factor  $(\lambda - 1 - 2\alpha)^{-2}$ , one may expect that the three-spin interaction can dramatically change the GP modification around the point  $\lambda - 1 - 2\alpha = 0$  except  $\beta = 0, \pi/2$  and  $\pi$ . Meanwhile the GP modification is positive when  $\beta < \pi/2$  and negative when  $\beta > \pi/2$ , and changes sign around  $\beta = \pi/2$ .

In order to get the entire picture about the effect of the three-spin interaction on the GP, we have numerically investigated the variation of GP with various parameters of the spin chain according to Eq. (23).

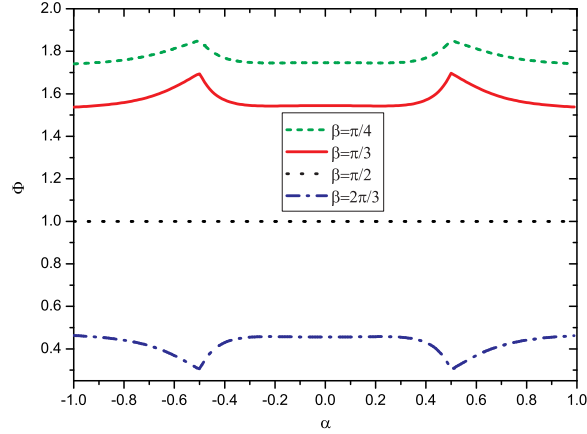


Figure 1: The GP versus the three-spin interaction strength  $\alpha$  for the different initial states in the Ising limit ( $\gamma = 1$ ) and without the external field ( $\lambda = 0$ ). The other parameters are chosen as  $N = 501, g = 0.03$  and  $\eta = 2\pi/3$ .

When the transverse magnetic field is absent, i.e.  $\lambda = 0$ , Eq. (30) shows that the GP modification may dip or peak around the point  $\alpha = -0.5$ , depending on the initial state of the qubit. In Fig. 1, the GP is plotted as a function of the three-spin interaction strength  $\alpha$  for the different initial states in the Ising limit ( $\gamma = 1$ ). The features of the GP on the parameter  $\beta$  shown in Fig. 1 are same as expected from Eq. (30). For a fixed value of  $\beta$ , except the points  $\beta = 0, \pi/2$  and  $\pi$ , the GP displays a peak or dip at the critical

point  $\alpha = -0.5$ . Moreover, it is noted that the GP also has a peak or dip at the critical point  $\alpha = 0.5$  which can not be expected from Eq. (30). In Eq. (18), the variable  $k$  takes values from 0 to  $N/2$ . Eqs. (11)-(12) depend on the variable  $k$  through the trigonometric functions  $\sin(2\pi k/N)$  and  $\cos(2\pi k/N)$ . Thus, Eqs. (11)-(12) must have dips or peaks around  $k = N/2$  if it has dips or peaks around  $k = 0$ . It is this reason that leads to the appearance of peaks or dips of the GP at  $\alpha = 0.5$ . Fig. 1 also shows that the critical point position does not change with variation of  $\beta$ , and is determined only by the three-spin interaction.

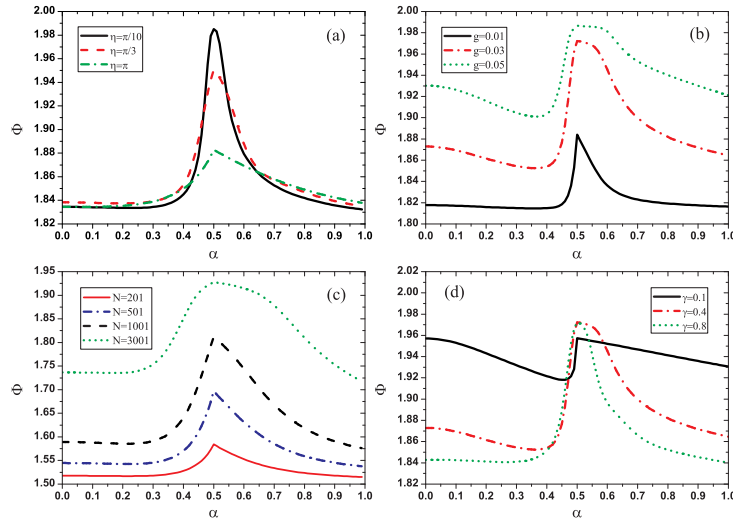


Figure 2: The GP versus the three-spin interaction  $\alpha$  with (a) different values of the transition frequency  $\eta$  and  $N = 501$ ,  $g = 0.03$ ,  $\beta = \pi/5$  and  $\gamma = 1$ ; (b) different values of the qubit coupling strength  $g$  and  $N = 501$ ,  $\eta = \pi/5$ ,  $\beta = \pi/5$  and  $\gamma = 0.4$ ; (c) different values of the spin chain size  $N$ , and  $g = 0.03$ ,  $\eta = 2\pi/3$ ,  $\beta = \pi/3$ ,  $\gamma = 1$  and (d) different values of the anisotropic parameters  $\gamma$ , and  $g = 0.03$ ,  $N = 501$ ,  $\eta = \pi/5$ ,  $\beta = \pi/5$ .

In Figs. 2, the GP is plotted as a function of  $\alpha$  with various values of the transition frequency  $\eta$ , the coupling strength  $g$  and the spin chain size  $N$  for the case of  $\lambda = 0$ , respectively. The GP versus  $\alpha$  is shown in Fig. 2(a). It is observed that the variation of the GP is sharpened around  $\alpha = 0.5$  by decreasing the transition frequency  $\eta$ , which is in accordance with the analytical result of Eq. (30). The influence of the coupling strength  $g$  on

the GP is shown in Fig. 2(b). Obviously, the peaks of the GP become less pronounced with increasing  $g$ . It means that as a tool of detecting QPTs the GP is powerful only in the weak coupling case. The influence of the spin chain size  $N$  on the GP is given in Fig. 2(c). It is seen that the peak becomes less sharp with increasing  $N$ . The GP of the qubit coupled to the XY spin chain ( $\gamma \neq 1$ ) is shown in Fig. 2(d). Obviously, the larger the anisotropy parameter is, the more sharp the GP peak is.

Combing Fig. 1 with Fig. 2, we see that by varying the parameters the peak values of the GP is changed but the peak position of the GP is retained. Thus, the QPT at  $|\alpha| = 0.5$  induced by the intrinsic three-spin interaction can be clearly singled out by the GP of the central qubit.

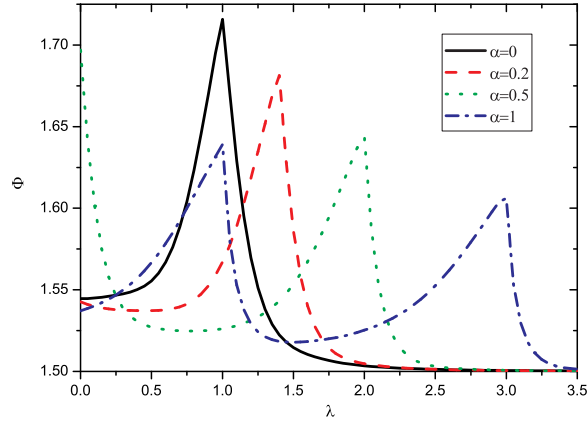


Figure 3: The GP as a function of  $\lambda$  with different values of the three-spin interaction strength  $\alpha$  in the Ising limit. The other parameters are chosen as  $\gamma = 1$ ,  $N = 501$ ,  $g = 0.03$ ,  $\eta = 2\pi/3$  and  $\beta = \pi/3$ .

In Fig. 3, the GP is shown as a function of  $\lambda$  with various values of the three-spin interaction strength  $\alpha$  in the Ising limit. When the three-spin interaction is absent, i.e.  $\alpha = 0$ , the spin chain has a ground-state phase transition at  $\lambda = 1$ . As shown in Fig. 3, the GP displays a peak at  $\lambda = 1$  when  $\alpha = 0$ . When the three-spin interaction is switched on ( $\alpha \neq 0$ ), as expected from (30), the three-spin interaction changes the original critical point and the GP peak has a  $2\alpha$  displacement from the point  $\lambda = 1$ .

In Figs. 4, the GP is plotted as a function of  $\lambda$  with  $\alpha = 0.2$  for different

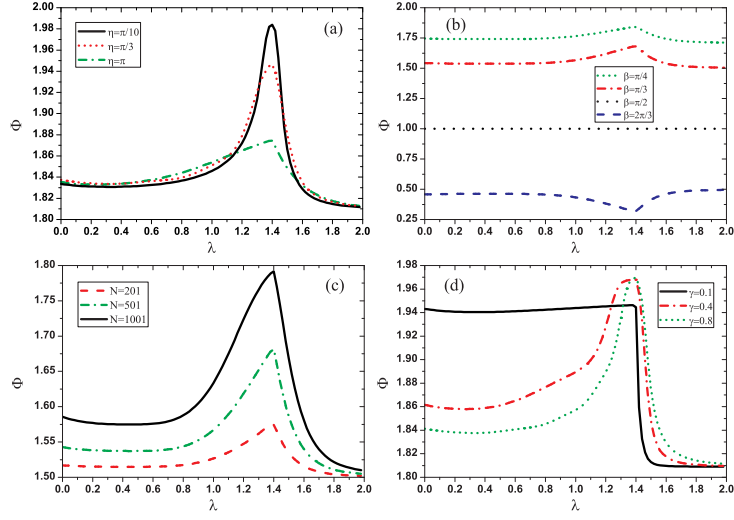


Figure 4: The GP versus the coupling constant  $\lambda$  for  $\alpha = 0.2$  with (a) different values of the transition frequency  $\eta$  and  $N = 501, g = 0.03, \beta = \pi/5$  and  $\gamma = 1$ ; (b) different values of  $\beta$  and  $N = 501, \eta = 2\pi/3, g = 0.03$  and  $\gamma = 1$ ; (c) different values of the spin chain size  $N, g = 0.03, \eta = 2\pi/3, \beta = \pi/3, \gamma = 1$  and (d) different values of the anisotropic parameters  $\gamma$ , and  $g = 0.03, N = 501, \eta = \pi/5, \beta = \pi/5$ .

values of the transition frequency  $\eta$ , the initial state coefficient  $\beta$ , the spin chain size  $N$  and the anisotropy parameter  $\gamma$ . In the Ising limit, by setting  $N = 501$ ,  $g = 0.03$ ,  $\gamma = 1$  and  $\beta = \pi/5$ , the variation of the GP is shown against  $\lambda$  in Fig. 4(a). It is observed that the GP peaks have a 0.4 displacement from  $\lambda = 1$  and become more sharp around  $\lambda = 1.4$  with decreasing  $\eta$ , much similar to that in Fig. 2(a). By choosing  $N = 501$ ,  $\eta = 2\pi/3$  and  $g = 0.03$ , Fig. 4(b) shows the influence of  $\beta$  on the GP. It can be seen that the GP peaks also move to the point  $\lambda = 1.4$  but the peak value decreases with increasing  $\beta$ . The influence of the spin chain size  $N$  on the GP is shown in Fig. 4(c) with the parameters  $g = 0.03$ ,  $\eta = 2\pi/3$  and  $\beta = \pi/3$ . It is seen that the GP peak value increases with increasing  $N$ . In Fig. 4(d), the GP is plotted as a function of the anisotropy parameter with  $N = 501$ ,  $\eta = \pi/5$  and  $\beta = \pi/5$ . It is clear that the GP peak becomes pronounced but the critical point is not shifted as the anisotropy parameter increases.

#### 4. Summary

We investigate geometric phase (GP) of a qubit symmetrically coupled to an anisotropic XY spin chain with three-spin interaction in a magnetic field. An analytical expression for the GP is obtained in the weak coupling limit. We find that the GP displays a peak or dip at the quantum phase transition (QPT) points of the spin chain, depending on the initial state of the qubit. Without the three-spin interaction, the QPT appears at the critical point  $\lambda = 1$  and the GP has a peak or dip around this critical point. If the three-spin interaction exists, the peak or dip position of the GP is obviously shifted away from the original position ( $\lambda = 1$ ). Moreover, it is analytically and numerically shown that the position shift of the GP peak or dip is determined only by the three-spin interaction. Thus, it comes to the conclusion that the GP can be used as a tool to detect both the existence and strength of multi-spin interaction in a spin chain.

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